Conservation of momentum

I. Newton’s laws can be unified to Conservation of momentum
II. Momentum is \( p = mv \) where \( m \) is the mass and \( v \) is the velocity
III. In a closed system \( p \) is invariant!!!
IV. Example: Show balloon
   A. Rocket weighing \( m_r \) with fuel weighing \( m_f \) at rest
   B. Draw rocket.
   C. shoots the fuel out at some velocity \( v_f \)
   D. The final velocity is given by conservation of momentum.

\[
p_0 = p_{\text{final}} \Rightarrow 0 = m_f(-v_f) + m_r(v_r) \Rightarrow v_r = m_fv_f/m_r
\]
E. In balloon:
   1. \( m_{\text{air}} = 1.3 \text{ gm/liter} \) \( m_{\text{balloon}} = 0.7 \text{ gm} \)
   2. \( v_r = 1.3 \text{ gm} v_f/0.7 \text{ gm} \)
   3. Why doesn’t this work very well? Air friction!
F. In a rocket which weighs more the rocket or the fuel? Try to make \( m_f \) large and \( m_r \) small to make \( v_r \) high
V. Works in relativity though the definition of \( p \) changes.
VI. Connection to \( F = ma \)
   A. Force is defined as a change in momentum per unit time.
   B. \( \frac{dp}{dt} = \frac{dmv}{dt} = m\frac{dv}{dt} = ma \)
VII. Other Examples? Gun?
VIII. Angular momentum
   A. All bodies conserve linear momentum, \( p = mv \), then they conserve angular momentum too.
   B. angular momentum, \( L = mvr \), where \( m \) is the mass, \( v \) is the spin velocity and \( r \) is the distance from some fixed point (usually the center of the spin).
   C. \( L \) is conserved unless an external force is at work.
   D. An external force is called a \textit{torque}.
   E. Classic example: Ice skater pulling in hands.
   F. Demo
   G. Useful in many calculations
H. Shows Kepler’s second law
   1. Equal areas in equal time
   2. Earth’s orbit:
      a. Perihelion 0.98AU
      b. Aphelion: 1.017AU
      c. If we know velocity at Perihelion = \( v_p \) then \( L = m_E v_p D_p \)
      d. Gives us the velocity everywhere since \( L \) is conserved
      e. What is the velocity at Aphelion?

\[
L = m_E v_a D_a = m_E v_p D_p \Rightarrow v_a = v_p D_p / D_a = v_a 0.96
\]

Universal Law of gravity

I. Principia goes on to use the laws of mechanics plus a new Law of Gravitation to unify Kepler's three laws and more...

II. Force between 2 bodies is directly proportional to their masses and inversely proportional to the square of the distance between them.

III. Show figure

IV. 

\[
F = \frac{G m_1 m_2}{d^2}
\]

V. \( G = 6.67 \times 10^{-11} \text{ m}^3 / \text{kg} \text{ s}^2 \)

VI. Force between a student and me:

A. 

\[
F = \frac{6.67 \times 10^{-11} \text{ m}^3 / \text{kg} \text{ s}^2}{1 \text{ m}^2} \cdot \frac{80 \text{ kg} \times 50 \text{ kg}}{1 \text{ m}^2} = 2.7 \times 10^{-8} \text{ kg m s}^2
\]

B. Is this large or small?

C. Force of gravity on me \( F = 80 \text{ kg} \times 9.8 \text{m} / \text{s}^2 = 784 \text{ N} \)

D. So \( F_s / F_g = 3E - 12 \)

VII. Force Between the Earth and me:

A. We know the masses \( M_{\text{earth}} = 6.0 \times 10^{24} \text{kg} \)

B. How far away is the Earth?

1. I’m standing on it so \( d = 0 \Rightarrow F = \infty \). Wrong!

2. Have to add up all the pieces

3. Diagram of man standing on Earth with each piece pulling him.
4. This difficulty required the invention of calculus and partially caused the 20 year delay in publishing Principia.
5. Turns out one goes through all those calculations and the result is the same as if all the mass were at the center of the Earth.
6. d=6,400km

C.

\[ F = \frac{6.67 \times 10^{-11} \frac{m^3}{kg^2} \cdot 80 \ kg \cdot 6.0 \times 10^{24} \ kg}{(6.4 \times 10^6 \ m)^2} = 780 \ \frac{kg \ m}{s^2} \]

D. Familiar?

VIII. What if I were in the shuttle?

A. How high up? 200km
B. d=6,400km+200km=6,600km
C.

\[ F = \frac{6.67 \times 10^{-11} \frac{m^3}{kg^2} \cdot 80 \ kg \cdot 6.0 \times 10^{24} \ kg}{(6.6 \times 10^6 \ m)^2} = 735 \ \frac{kg \ m}{s^2} \]

D. \( F = ma \Rightarrow a = F/m = (735 \ kg \ m/s^2)/(80 \ kg) = 9.2 m/s^2 = 0.94g \)
E. Aren’t the space-men in the shuttle weightless?
F. Weightless means relative weightlessness. They are falling together

G. Elevator example.

1. If an elevator cord snaps how much will you weigh?
2. If an elevator drops, accelerating at 4.9m/s/s, how much will you weigh?
3. If an elevator ascends, accelerating at 9.8m/s/s, how much will you weigh?

IX. Orbits

A. Trapped orbits:

1. Can you name examples: Earth, moon, Sun around Milky-way galaxy - almost everything astronomical.
2. All these objects are falling but their momentum carries them around the Earth

B. Untrapped orbits: hyperbolic and parabolic
X. Before the laws of mechanics and motion, a cannon could be aimed through trial and error. Now we can shoot an automated projectile at the moons of Jupiter and hit! Alexander Pope wrote upon Newton’s Death in 1727:

God said, Let Newton be! and all was light.

XI. Newton’s Madness: A story of genius and madness
A. Twice during his life 1677-1678 and 1693
B. After 1678 he withdrew from intellectual life for 6 years (his mother also died in 1679).